RECOMMENDED EXERCISES – CHAPTER 1

1ª classroom pratice:

1. A box contains 5 balls of which 2 are black. The black balls are numbered 1 and 2, the others 3 to 5. Two balls are randomly taken out one after the other without replacement. The numbers on the two balls are observed.
    a. Enumerate all the elements of the sample space associated to this random experiment.
    b. Define in the sample space the following events:

       A_1 – The first ball observed is black.

       A_2 – The second ball observed is black.

       A_3 – The two balls taken out are black.

       A_4 – At least one of the two balls are black.

       A_5 – Exactly one of the two balls are black.

       A_6 – The sum of the numbers in the two balls is greater than seven.
    c. Make a chart of the sample space and the events defined above.

2. Two light bulbs will be kept on until both turned off. The life time of both bulbs are registered. None of the light bulbs have a life time greater than 1600 hours. Make a chart of the sample space and the following events:

   A – None of the light bulbs has a life time longer than 1000 hours.

   B – Just one of the bulbs has a life time greater than 1000 hours.

   C – The life time of one of the bulbs doubles the life time of the other.

   D– The sum of the life times of the two bulbs is greater than 2000 hours.
3. Define and classify the sample spaces associated with the following random experiments:
    a. Observation of the number of spots when a six face die is thrown.
    b. A coin is tossed and the face-up is observed.
    c. A die is Thrown followed by the toss of a coin.
    d. A coin is tossed until a head comes.

4. In a College, 70% of the students have a computer at home, 40% have a PC and 30% have both. If a student is randomly chosen, evaluate the probability of the student:
    a. Has at least one of the two types of computers.
    b. Has no computer.
    c. Has one of the two types of computers.
5. Weather forecast says that it will rain next Saturday with probability 0.25 and that it will rain next Sunday with probability 0.25. Can one say that according to the weather forecast the probability of rain next weekend is 0.5?
6. An electronic system is composed of two sub-systems, A and B. From previous rehearsals it is known that : the probability of A failure is 0.2, the probability that only B fails is 0.15 and the probability that A and B fail simultaneously is 0.15. Evaluate the probability that:
    a. B fails.
    b. A fails alone.
    c. Fail at least one of them, A or B.
    d. Neither A nor B fails.
    e. A and B don't fail simultaneously.
7. The workers of a firm regularly use public transports (bus, metro, train) to go from home to work. It is known that:
    - 54% use exclusively one of these public transports (bus-22%, metro-25%, train-7%);
    - 44% use at least two of the three public transports: 18%use bus and metro; 17% use bus and train; 19% use metro and train.
    a. Having in mind that there are other means of transport beyond the above mentioned, calculate the percentage of workers that don't use any of the above mentioned means of transport.
    b. Calculate the percentage of workers that use all the above mentioned means of transport.

8. For each of the following cases designate, justifying, the adequate definition of probability to evaluate the probability that:
    a. Next year inflation rate be greater than 5%.
    b. Observe 6 spots after throwing a six face regular die.
    c. A piece taken randomly of a numerous lot be defective.
    d. Getting a first prize in the lottery.
    e. The GDP growth rate next year is greater than 3%.
    f. A randomly chosen individual from those entering a mall buy something.
9. Take the succession of natural numbers, 1, 2, $\cdots, n$, and chose two randomly without replacement. Evaluate the probability of one of them be lower than k $(k = 1, 2, 3, \cdots, n-1)$ and the other greater than k.
10. In a classroom there are 30 students, 20 boys and 10 girls. Four students are selected to form a committee representing the class.
    a. Calculate the probability that the first two selected are boys and the next two girls.
    b. What is the probability that the committee has two girls and two boys?
    c. What is the probability that the first student selected is a boy? And the third?
11. Suppose that a CD has 14 songs and you like 8 of them. Pressing a button of random selection of the CD player, each of the 14 songs will play once by a random order. What is the probability that, from the two first songs played you liked just one.
12. Mastermind is a game where the 1st player choose 4 pins with different colours (6 colours available for each pin) and stick them on a board by a certain order hidden from the 2nd player. Evaluate the probability that the 2nd player guess the hidden key at first attempt.
13. Consider a computer system that generates randomly a key-word for a new user composed of 5 letters (eventually repeated) of an alphabet of 26 letters (no distinction is made between capital and lower case letters). Calculate the probability that there is no repeated letters in the key-word.

14. The Loto game consists of extracting 6 balls without replacement from an urn with balls numbered from 1 to 49. A 7^{th} ball is extracted for the 2^{nd} prize. Each bet corresponds to a choice of 6 numbers.
    a. What is the probability of the 5^{th} prize (hitting 3 of 6 numbers randomly selected)
    b. If you hit 5 of 6 numbers randomly selected plus the one in the 7^{th} ball you get the 2^{nd} prize. Calculate the probability of such event.
15. The Euromillions game consists of extracting 5 balls without replacement from an urn with balls numbered from 1 to 50 and two stars numbered 1 to 9. Each bet corresponds to a choice of 5 numbers and 2 stars. Evaluate the probability that:
    a. You win a 1^{st} prize (hitting 5 numbers and 2 stars).
    b. You win the last prize (hitting 2 numbers and 1 star).
16. Consider a multiple response test with 20 questions, each with 4 possible answers. Supposing that each question values 1 point and the wrong answers are not negatively valued, if the student answers randomly evaluate the probability that the student:
    a. Get all answers wrong.
    b. Has a mark greater than 9. Formalize.
17. Consider a box with 20 balls – 7 black, 5 white and the others yellow.
    a. Two ball are extracted without replacement. What is the probability that at least one is black?
    b. If 6 balls are extracted without replacement, what is the probability that there are two of each colour? Evaluate the probability of the same event if the extraction is with replacement.
    c. A random experiment consist of throwing a regular die and then take out of the box ,without replacement, a number of balls equal to the number of spots on the die. Calculate the probability that all extracted balls are black.
18. In a learning experience, an individual realizes twice consecutively a task where he can fail or be successful in any of them. The probability of failing the first attempt is 0.25. If he fails the first attempt the probability of being successful in the second is 0.5. If he is successful in the first attempt, the probability of failing in the second is 0.1. What is the probability of failing the second attempt.

19. In a recreational tournament between societies A and B, three basketball games take place with three teams of both societies: A_1 against B_1; A_2 against B_2; A_3 against B_3. Society A win the games with probability, 0.8, 0.4 and 0.4 respectively. Knowing that there are no draws in basketball games and that to win the tournament , at least 2 games from the three has to be won, which of the societies is the favorite.
20. In order to fill a 90 minute period television broadcast, two one hour programs – one musical and the other sports – and three half an hour programs – one information and two musicals- are available to build a program schedule (the order is relevant).
    a. Choosing randomly a program schedule, what is the probability that it is composed only with musicals?
    b. In 10% of the program schedules composed only of half an hour programs there are transmission problems. Only, 5 % of the other program schedules have the same problem. Evaluate the probability that a randomly selected program schedule has such a problem.
21. A factory uses three machines to produce the same product. Machines A, B and C produce respectively 40%, 35% and 25% of total production. The percentage of defective parts produced by each machine are respectively 4%, 2% and 1%. If a piece is randomly selected from the total production.
    a. What is the probability that it is not defective?
    b. Knowing that it is defective what is the probability that it has been produced by machine A?
    c. If two pieces are successively removed with replacement from the total production, what is the probability that one of them be defective and the other not?

22. The employees of a certain firm were classified in three training degrees: low, medium, high. It is known that:
    - 55% of the employees have a wage greater than 1000 euros;
    - 40% of those with medium training degree have a wage greater than 1000 euros;
    - 70% of those with high training degree have a wage greater than 1000 euros;
    - None of the employees with low training have a wage greater than 1000 euros;
    - There are 10% of employees with low training degree.

    a. Evaluate the probability that a randomly chosen employee has an high training degree.
    b. Calculate the probability that an employee has an high training degree, if he earns a wage greater than 1000 euros.

23. A firm has 3 assembly lines, $A_{1,}$ $A_{2,}$ A_3, to produce a certain product. 5% of products from Line A_1 are defective. 8% of products from Line A_2 are defective and 10% of products from Line A_3 are defective. Knowing that line A_1 produces 50% and line A_2 30% of the total production, what is the probability that a random chosen product is defective? If a random chosen product is defective what is the probability that it comes from each one of the assembly lines?
24. The two more important suppliers of eggs of a supermarket - F_1 e F_2- supply respectively 50% and 40% of the total eggs bought by the supermarket. Some of the eggs are rotten. It is known that 60% of the rotten eggs come from $F_1$and 30% from F_2. It is also known that 5% of the rotten eggs come from other suppliers.
    a. What is the percentage of rotten eggs received by the supermarket?
    b. What is the percentage of rotten eggs supplied by F_1?
    c. If 6 eggs are selected randomly, what is the probability that 3 of them have been supplied by F_2?

25. To select a person to a certain service, each candidate is submitted successively to three tests – A, B and C. They follow to the next test if they get at least 60% in each test. The candidates are selected if they get at least 60% in each test or at least 90% in some test. The candidates are submitted to the tests one by one, and the process stops if any of them is selected. From the experience it is known that:
    - From those who did test A, 10% have at least 90%, 30% have between 60 and 90%;
    - From those who proceed to test B, 20% have at least 90%, 40% have between 60 and 90%;
    - From those who proceed to test C, 50% have at least 60%.

    Evaluate the probability that they have to analise two candidates to select a person?
26. On the journey of a war plane there are two enemy radar stations equipped with anti-aircraft guns, which are triggered only if the plane is detected. The plane has a 0.25 chance of being detected by the 1st station, and, being detected, has three chances in five not to be shot down by that station. If the plane is not detected by the 1st station, it approaches the 2nd under the same conditions as the first. On the other hand, if the first station detects it without shooting it, it will certainly be detected and shot down by the 2nd station.
    a. What is the probability that the plane would not be shot down for the 1st station?
    b. What is the probability that the plane would not be shot down?
    c. Knowing that the plane has been shot down, evaluate the probability that it has been shot down by the 1st station?
27. The probability of a person of a given city being diabetic is 0.02. The test used to detect the disease gives positive result in 90% of diabetics and in 5% of non-diabetics.
    a. What is the likelihood that the test be positive for one person chosen at random?
    b. Knowing that the test result is positive, what is the probability that the individual is a diabetic?

28. Certain virus, for which there is a treatment that allows you to control its evolution, affects some individuals in adulthood. The disease manifests itself in severe form in 2% of cases, 12% in a moderate form, and does not affect the other. The test used to detect the disease gives positive result in 95% of serious cases, in 80% of moderate cases and in 5% of healthy individuals.
    a. What is the probability that the test be positive for a random chosen individual?
    b. Show that the majority of the time in that the test is positive, the person is moderately affected by the virus.
    c. If 100 individuals were chosen at random, what is the probability that 3 of them are seriously affected by virus?
29. Suppose one cast a regular die 3 times. Event A occurs if one got 1 or 2 in the first cast. Event B occurs if one got 3 or 4 in the second cast. Event C occurs if one got 5 or 6 in the third cast. Find $P(A \cup B \cup C)$.
30. Events A, B and C constitute a partition of the sample space Ω with positive probabilities. Show that if event $E \subset (B \cup C)$, then $P(E) = P(\bar{A}).P(E|\bar{A})$.
31. Let A and B be independent events of the same sample space. If $P(A) = \frac{1}{3}$ e $P(B) = \frac{3}{4}$,
    a. Calculate $P(A \cup B)$ and $P(B|A \cup B)$.
    b. Show that the complement of A is independent of the complement of B too.
32. Given events A and B, show that $P(A \cap B) \leq P(A) \leq P(A \cup B)$.
33. Consider Events A, B and C of the same sample space. Suppose that: the probabilities of such events are positive; $A \subset C$; events B and C are mutually exclusive events. Show that

$$P[(A \cup B)|C] = \frac{P(A)}{P(C)}.$$

34. Suppose that you have a friend who had never studied statistics. Using illustrative examples explain the difference between mutually exclusive events and statistically independent events.
35. Prove that, if $P(B) > 0 \Rightarrow P(A \cup B|C) = P(A|B) + P(C|B) - P(A \cap C|B)$.
36. Prove that, if $P(A|E) \geq P(B|E)$ e $P(A|\bar{E}) \geq P(B|\bar{E})$ then $P(A) \geq P(B)$.
37. Is it possible that one have a situation where $P(A) = 1/4\,, P(B) = 1/2$ and $P(A \cup B) = 1/3$? Please justify.